
A Predictive Control Perspective on Electricity Markets

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**UIUC Seminar
May 5th, 2011 (Cinco de Mayo)**



Outline

1. Motivation

Next-Generation Power Grid
Market Volatility and Instability

2. Predictive Control Framework

Market as Receding Horizon Game

3. Stability and Robustness

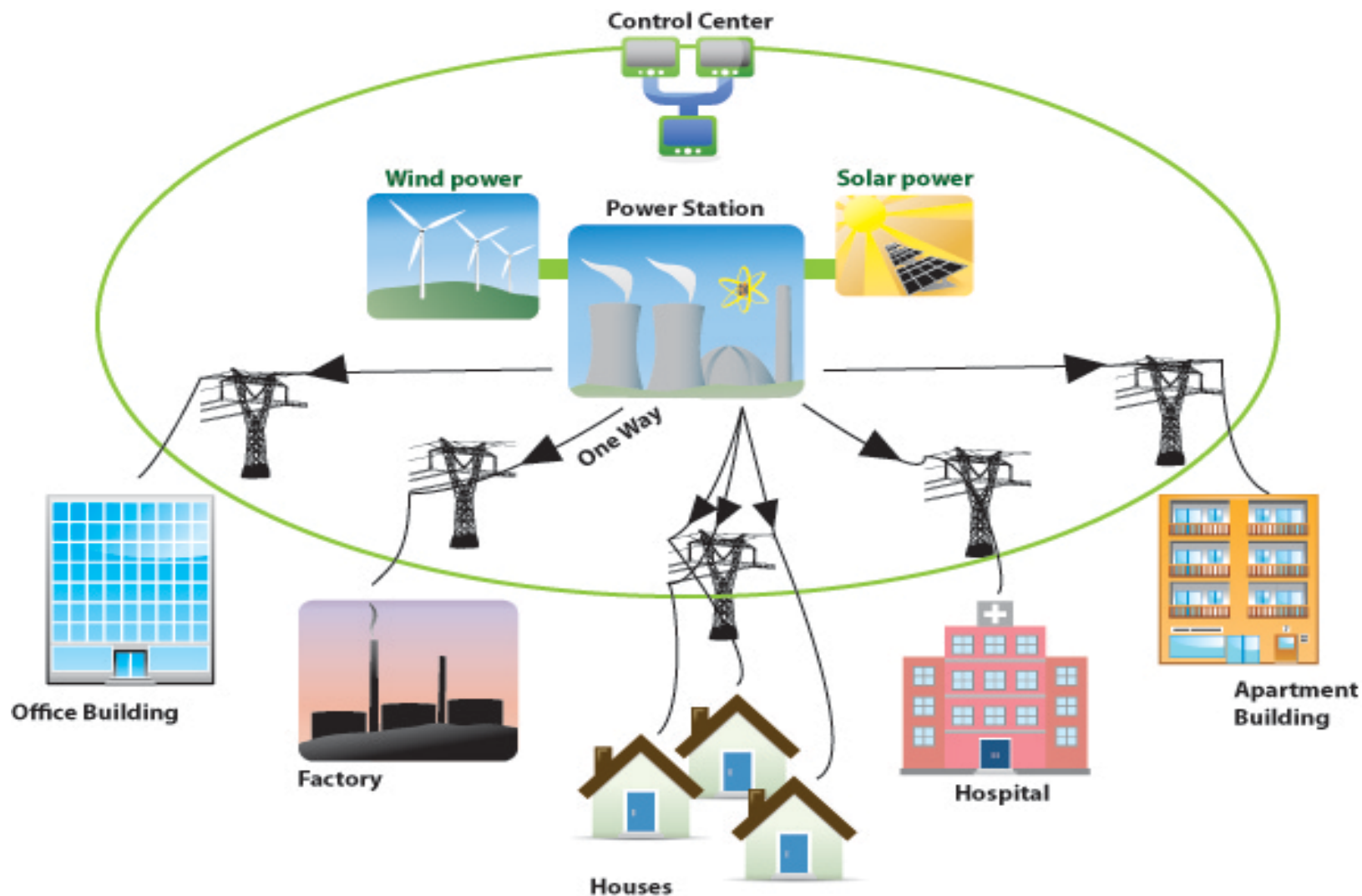
Finite Horizons, Incomplete Gaming, & Forecast Errors

4. Numerical Examples

5. Conclusions and Open Questions

1. Motivation

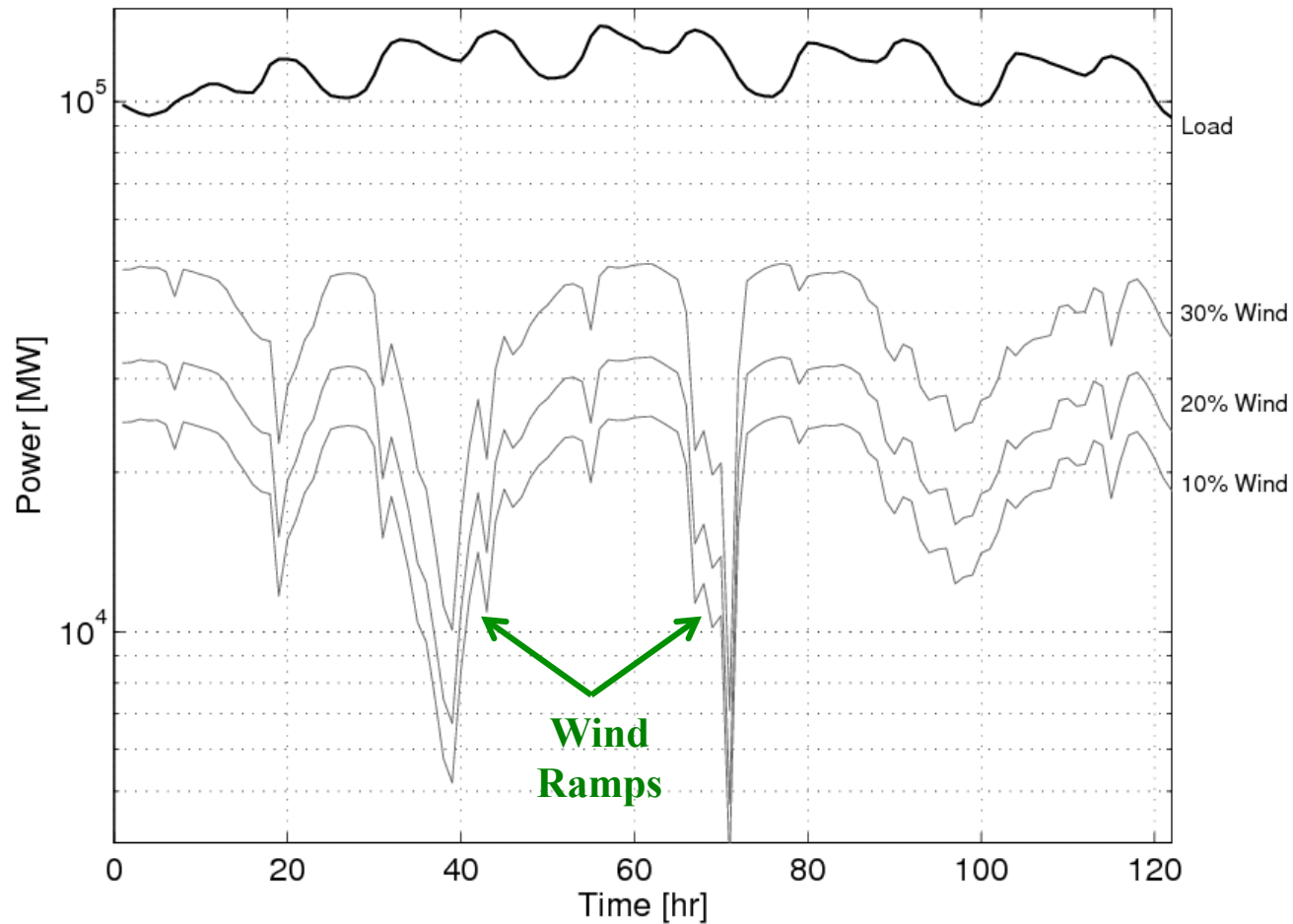
Current Grid



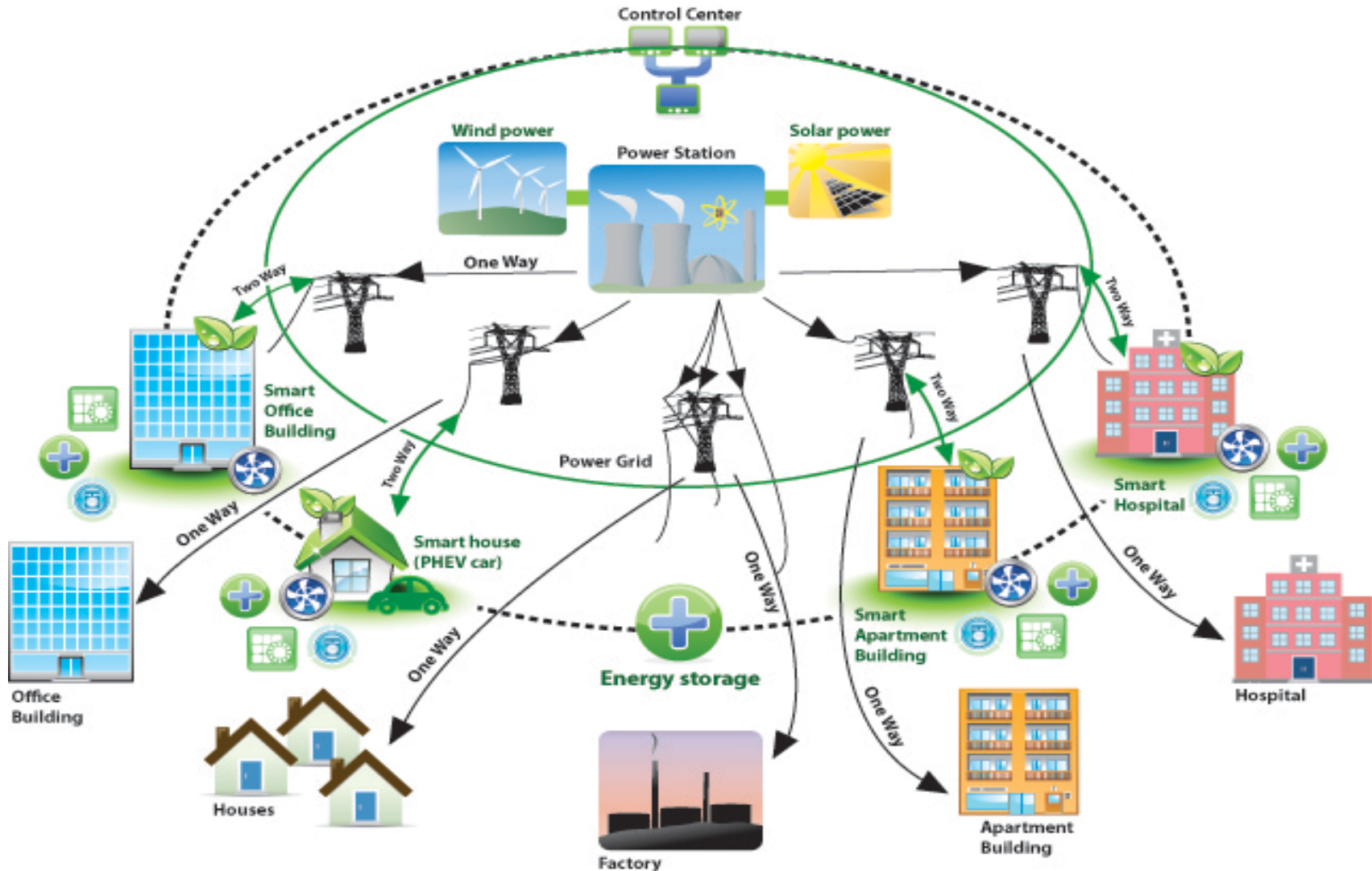
~ 70% Electricity from Central Coal Plants – CO₂ Emissions
Limited Market Control – Demands are Inelastic, No Storage, Slow Generation
Cannot Sustain High Renewable Supply -Intermittent-

Renewable Supply

Supply -Wind- and Elastic Demands Vary at Higher Frequencies



Next-Generation Grid

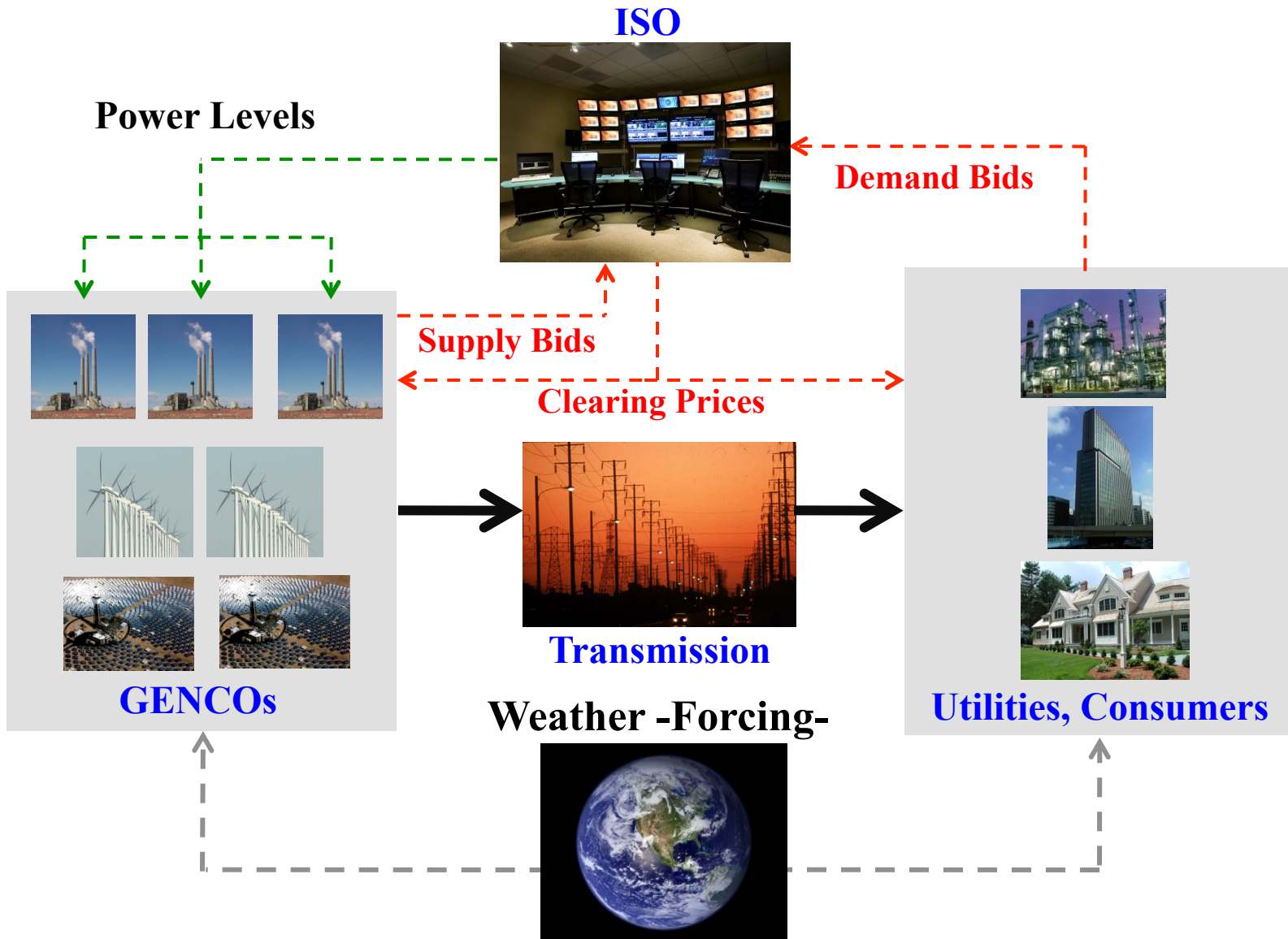


Major Adoption of Renewables -30%-

Real-Time Pricing + Demand Response - Elastic Demands-

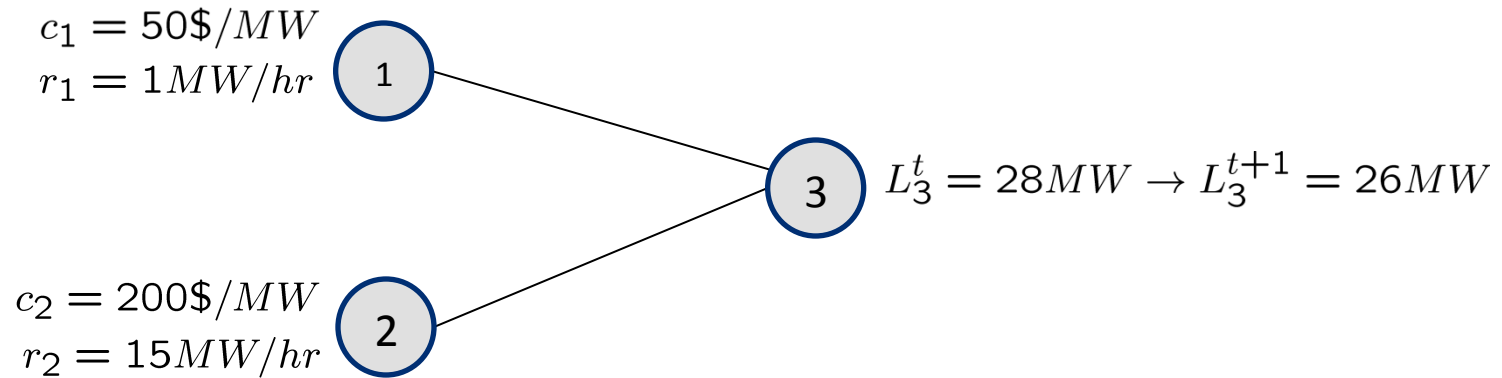
Huge Investments in Natural Gas Generation –Faster Response-

Electricity Markets



Dynamic & Uncertain Forcing Factors -Weather- Drive Markets
Volatility Due to Market Friction: (Capacity, Ramping, Congestion)

Market Instability and Ramp Constraints



No Ramp Constraints $\lambda^t = 50\$/MW(28, 0) \rightarrow \lambda^{t+1} = 50\$/MW(26, 0)$

Ramp Constraints (No Foresight)

$G_{t-1}^1 = 27 MW$
 $G_{t-1}^2 = 1 MW$

$\lambda^t = 50\$/MW(28, 0) \rightarrow \lambda^{t+1} = 0\$/MW(27, 0)$

Ramp Constraints (No Foresight)

$G_{t-1}^1 = 26 MW$
 $G_{t-1}^2 = 2 MW$

$\lambda^t = 50\$/MW(27, 1) \rightarrow \lambda^{t+1} = 50\$/MW(26, 0)$

Ramp Constraints (with Foresight)

$G_{t-1}^1 = 27 MW$
 $G_{t-1}^2 = 1 MW$

$\lambda^t = 55.35\$/MW(27, 1) \rightarrow \lambda^{t+1} = 50\$/MW(26, 0)$

Ramps Lead to Market Volatility – Propagation Through Initial Conditions (Need Foresight)

2. Predictive Control Framework

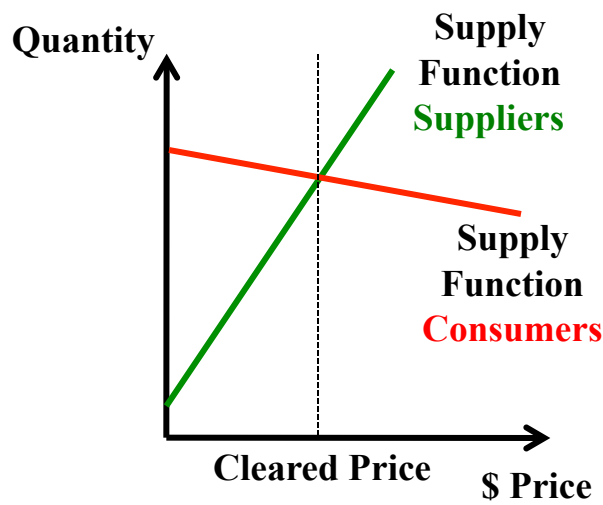
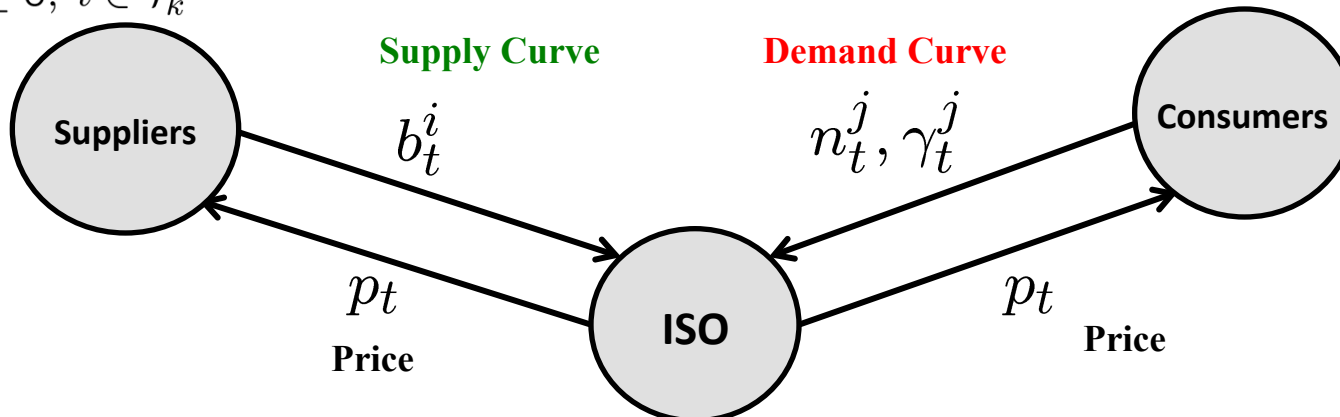
Predictive Control Framework

$$\max_{b_t^i, \Delta b_t^i} \sum_{t \in \mathcal{T}_k} (p_t \cdot b_t^i \cdot p_t - c_t^i(b_t^i \cdot p_t))$$

$$\text{s.t. } \underline{q}^i \leq b_t^i \cdot p_t \leq \bar{q}^i, t \in \mathcal{T}_k$$

$$b_t^i \geq 0, t \in \mathcal{T}_k$$

$$d_t^j = n_t^j - \gamma_t^j \cdot p_t$$



$$\min_{q_t^i, \Delta q_t^i} \sum_{t \in \mathcal{T}_k} \varphi_t := \sum_{t \in \mathcal{T}_k} \sum_{i \in \mathcal{S}} \int_0^{q_t^i} p_t(q, b_t^i) dq$$

$$\text{s.t. } q_{t+1}^i = q_t^i + \Delta q_t^i, i \in \mathcal{S}, t \in \mathcal{T}_k^-$$

$$\sum_{i \in \mathcal{S}} q_t^i \geq \sum_{j \in \mathcal{C}} d_t^j, t \in \mathcal{T}_k \quad (p_t)$$

$$-r^i \leq \Delta q_t^i \leq \bar{r}^i, i \in \mathcal{S}, t \in \mathcal{T}_k^-$$

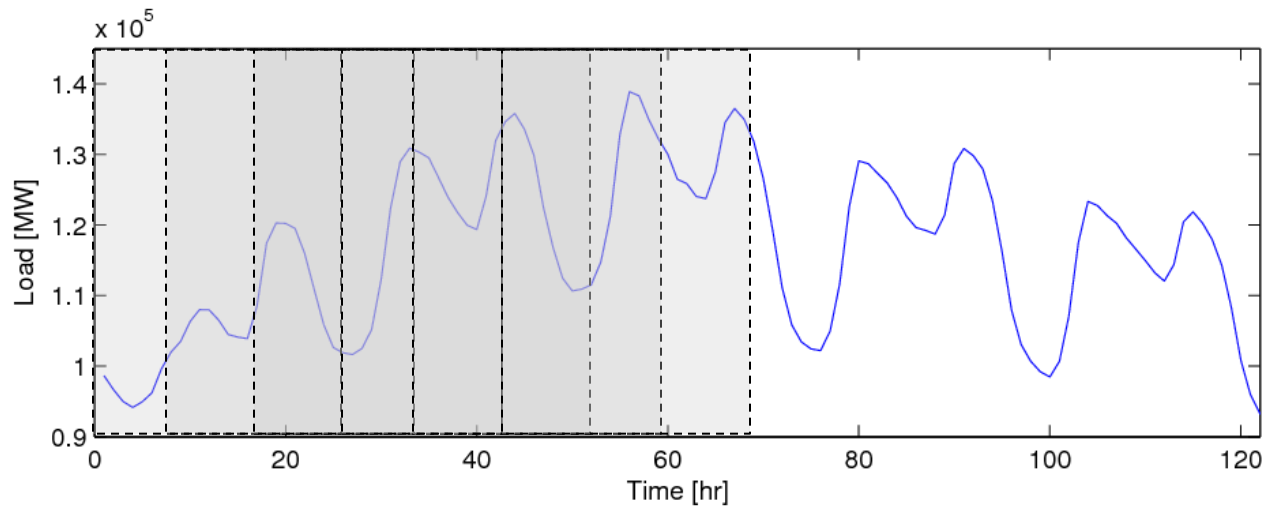
$$\underline{q}^i \leq q_t^i \leq \bar{q}^i, i \in \mathcal{S}, t \in \mathcal{T}_k$$

$$q_k^i = \text{given}, i \in \mathcal{S}.$$

Current Market Design : Game Runs Incompletely (Jacobi-Like Iteration)

Predictive Control Framework

Current Markets: Game Implemented Over Receding Horizon – Load



At k solve over $\mathcal{T}_k = \{k, \dots, k + T\} \Rightarrow$ Implement Price p_k

At $k + 1$ solve over $\mathcal{T}_{k+1} = \{k + 1, \dots, k + 1 + T\} \Rightarrow$ Implement Price p_{k+1}

Key Issues:

- How to Measure Dynamic Market Stability?
- Stability Under Finite Horizon
- Stability Under Incomplete (Suboptimal) Gaming
- Robustness Bounds
- Effect of Market Design: Frequency, Horizon, Strategic, Stabilizing Constraints
- Effect of Mechanistic Effects: Ramps, Topology, Congestion

3. Stability and Robustness

Market Stability (A Proposal)

Constrained Market Clearing

$$\begin{aligned}
 \min_{q_t^i, \Delta q_t^i} \sum_{t \in \mathcal{T}_k} \varphi_t &:= \sum_{t \in \mathcal{T}_k} \sum_{i \in \mathcal{S}} \int_0^{q_t^i} p_t(q, b_t^i) dq \\
 \text{s.t. } q_{t+1}^i &= q_t^i + \Delta q_t^i, \quad i \in \mathcal{S}, t \in \mathcal{T}_k^- \\
 \sum_{i \in \mathcal{S}} q_t^i &\geq \sum_{j \in \mathcal{C}} d_t^j, \quad t \in \mathcal{T}_k \quad (p_t) \\
 -\underline{r}^i &\leq \Delta q_t^i \leq \bar{r}^i, \quad i \in \mathcal{S}, t \in \mathcal{T}_k^- \\
 \underline{q}^i &\leq q_t^i \leq \bar{q}^i, \quad i \in \mathcal{S}, t \in \mathcal{T}_k \\
 q_k^i &= \text{given}, \quad i \in \mathcal{S}.
 \end{aligned}$$

Unconstrained Market Clearing (Marginal Cost)

$$\begin{aligned}
 \min_{q_t^i} \sum_{t \in \mathcal{T}_k} \varphi_t &= \sum_{t \in \mathcal{T}_k} \sum_{i \in \mathcal{S}} \int_0^{q_t^i} p_t(q, b_t^i) dq \\
 \text{s.t. } \sum_{i \in \mathcal{S}} q_t^i &\geq \sum_{j \in \mathcal{C}} d_t^j, \quad t \in \mathcal{T}_k \quad (\bar{p}_t) \\
 \underline{q}^i &\leq q_t^i \leq \bar{q}^i, \quad i \in \mathcal{S}, t \in \mathcal{T}_k,
 \end{aligned}$$

Property: For Fixed b_t^i , $\bar{\varphi}_t \leq \varphi_t, \forall t \in \mathcal{T}_k$

Definition: Market Efficiency. $\eta_t = \frac{\bar{\varphi}_t}{\varphi_t} \in [0, 1]$

Definition: Market Stability. The market given by the ISO/Supplier/Consumer game is stable if, given $\eta_0 \in \{\eta \mid \eta \geq \epsilon\}$ we have generation and demand sequences such that $\eta_t \in \{\eta \mid \eta \geq \epsilon\}, \forall t$.

Lyapunov Stability

Lyapunov Function = Indicator Function (Sufficient Conditions, Compare Designs)

Definition: Market Summarizing State.

$$\delta_{t+1} = \alpha(\eta_{t+1}, \epsilon) \cdot \delta_t \text{ with } \alpha(\eta, \epsilon) \leq 1 \text{ iff } \eta \leq \epsilon.$$

Observations: - Market Stability Implies Stability of Origin for Summarizing State
- Maximizing Efficiency Implies Minimizing Summarizing State

Abstract ISO Clearing Problem:

$$\begin{aligned} \min_{u_{\mathcal{T}_k^-}} \quad & \sum_{t \in \mathcal{T}_k^-} (\delta_{t+1} - \delta_t) \\ \text{s.t.} \quad & u_{\mathcal{T}_k} \in \Omega(\delta_k, d_{\mathcal{T}_k}) \\ & \delta_{t+1} = \alpha(\eta_{t+1}, \epsilon) \cdot \delta_t, \quad t \in \mathcal{T}_k^- \\ & \eta_t \geq \epsilon, \quad t \in \mathcal{T}_k \quad \leftarrow \text{Stabilizing Constraint} \\ & \delta_k = \text{given.} \end{aligned}$$

Candidate Lyapunov Function.

$$V_T(\delta_k, d_{\mathcal{T}_k}) := - \sum_{t \in \mathcal{T}_k^-} (\delta_{t+1} - \delta_t) = \delta_k - \delta_{k+T}.$$

Lyapunov Stability

Infinite Horizon: If game with horizon $T = \infty$ is feasible then, the market is stable.

Proof:

$$\begin{aligned}\Delta V_T(\delta_k) &= V_\infty(\delta_{k+1}, m_{\mathcal{T}_{k+1}}) - V_\infty(\delta_k, m_{\mathcal{T}_k}) \\ &= \sum_{t=k+1}^{\infty} (\delta_t^{k+1} - \delta_{t+1}^{k+1}) - \sum_{t=k}^{\infty} (\delta_t^k - \delta_{t+1}^k) \\ &= (\delta_{k+1} - \delta_\infty^{k+1}) - (\delta_k - \delta_\infty^k) \\ &= -(\delta_k - \delta_{k+1}) \\ &= (\alpha(\eta_{k+1}, \epsilon) - 1) \cdot \delta_k \quad \text{Accumulation Term} \\ &\leq 0\end{aligned}$$

Finite Horizon: Define Terminal Cost:

$$\Xi_k^1 := |V_T(\delta_{k+1}, m_{\mathcal{T}_{k+1}}) - V_{T-1}(\delta_{k+1}, m_{\mathcal{T}_k})|, \quad \Xi_k^1 \rightarrow 0, \quad T \rightarrow \infty$$

Finite Horizon: If game with horizon $T < \infty$ is feasible and the terminal cost is bounded by accumulation term, then the market is stable.

Proof:

$$\begin{aligned}\Delta V_T(\delta_k) &= V_T(\delta_{k+1}, m_{\mathcal{T}_{k+1}}) - V_T(\delta_k, m_{\mathcal{T}_k}) \\ &= (\alpha(\eta_{k+1}, \epsilon) - 1) \cdot \delta_k + \Xi_k^1 \\ &\leq 0\end{aligned}$$

Properties:

- Price Volatility Increases with Ramp Limits $\|p_t - \bar{p}_t\| \leq L(\|\bar{r} - \bar{q}\| + \|\underline{r} - \underline{q}\|)$
- **Key Outcome:** - Incomplete Game Cannot be Guaranteed to be Stable
 - Stabilizing Constraint “Filters Out” Suboptimal Bids

Robustness

Effect of Forecast Errors

Define Cost Perturbation:

Predicted State with Forecast

State with True Data

$$\Xi_k^2 := |V_T(\bar{\delta}_{k+1}, m_{\mathcal{T}_{k+1}}) - V_T(\delta_{k+1}, m_{\mathcal{T}_{k+1}})|.$$

Key: Boundedness of Perturbation Requires Game Numerical Stability

Numerical Stability: If at a solution of the game the players problems satisfy LICQ and the clearing prices are bounded away from zero, the game is stable and the solution is Lipschitz continuous on the data.

$$\max_{b_t^i, \Delta b_t^i} \sum_{t \in \mathcal{T}_k} (p_t \cdot b_t^i \cdot p_t - c_t^i (b_t^i \cdot p_t))$$

$$\text{s.t. } \underline{q}^i \leq b_t^i \cdot p_t \leq \bar{q}^i, \quad t \in \mathcal{T}_k$$

$$b_t^i \geq 0, \quad t \in \mathcal{T}_k$$

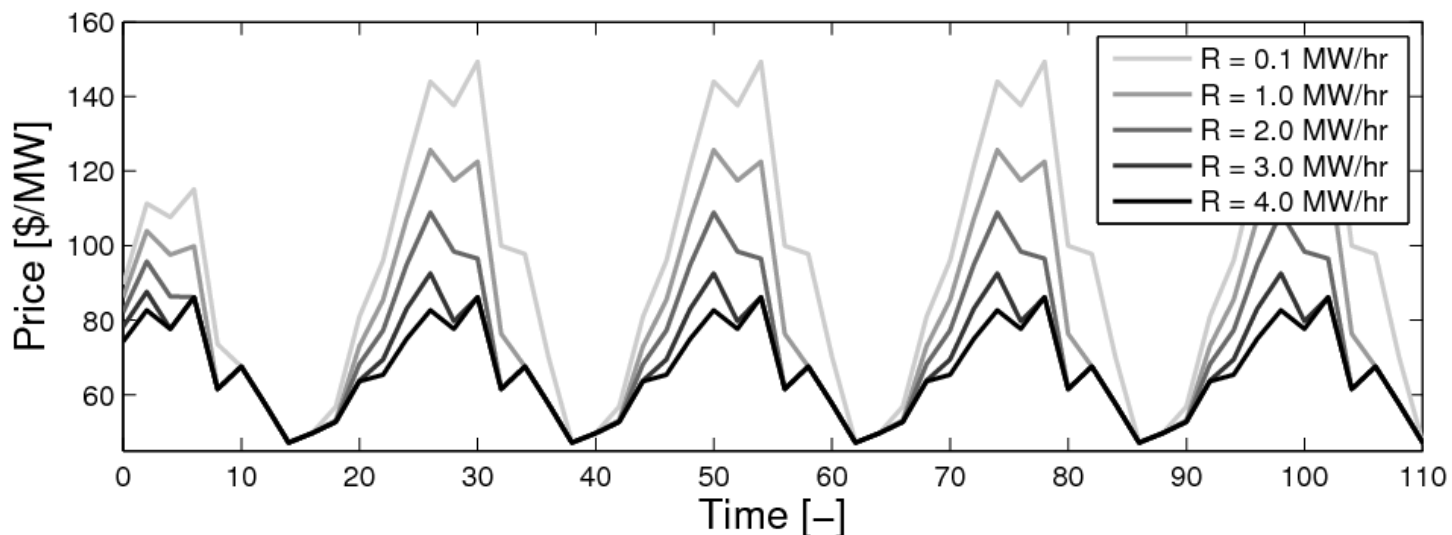
$p_t \rightarrow 0$ Destroys Curvature (Excess Supply)

Robust Finite Horizon: If game with horizon $T < \infty$ is feasible and the terminal cost and cost perturbation are bounded by accumulation term, then the market is stable.

Proof:

$$\begin{aligned} \Delta V_T(\delta_k) &= V_T(\delta_{k+1}, m_{\mathcal{T}_{k+1}}) - V_T(\delta_k, m_{\mathcal{T}_k}) \\ &= (\alpha(\eta_{k+1}, \epsilon) - 1) \cdot \delta_k + \Xi_k^1 + \Xi_k^2 \\ &\leq 0 \end{aligned}$$

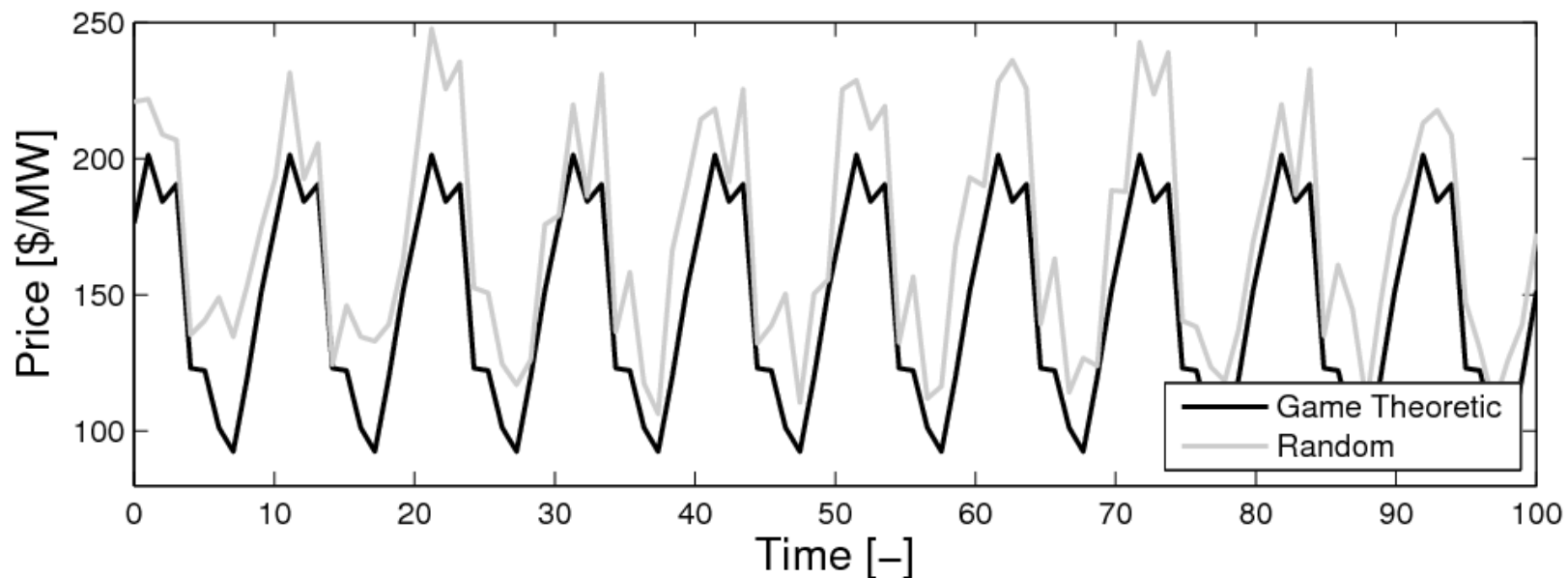
4. Numerical Examples



Dynamic Electricity Markets

Non-Gaming Behavior

Some Players -Intentionally or Unintentionally- Bid Suboptimally
Introduces Noise in Equilibrium – Can be Inferred from Data



Huge Potential for Dynamic Market Models

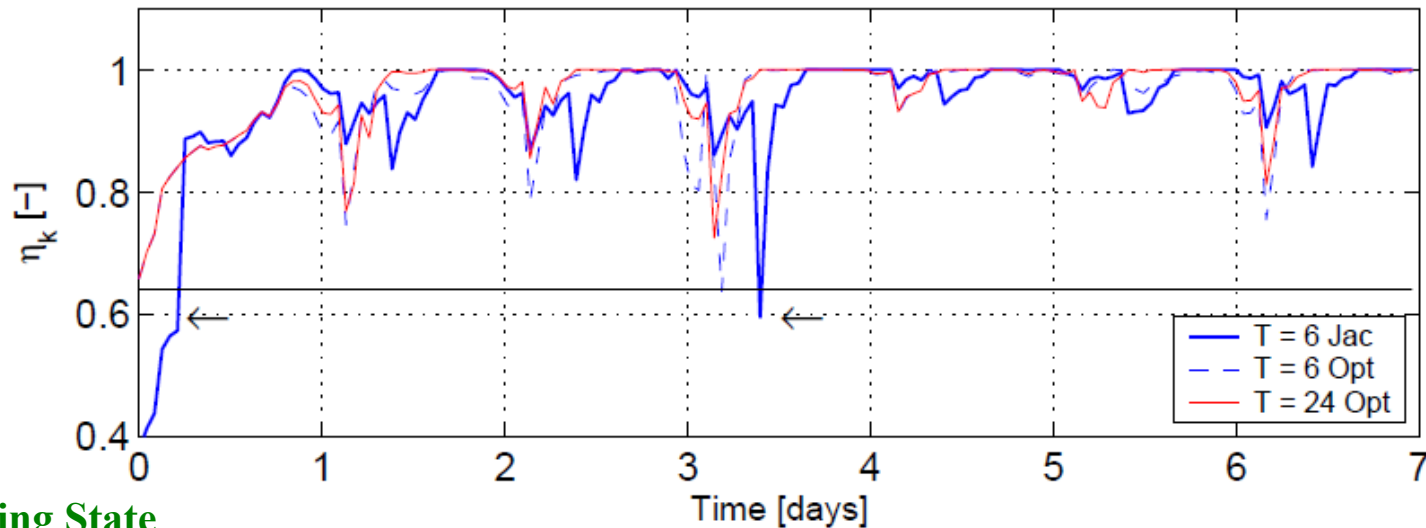
- Mechanistic Price Forecasting, Market Design and Monitoring
- Data Assimilation and State Estimation

Stability

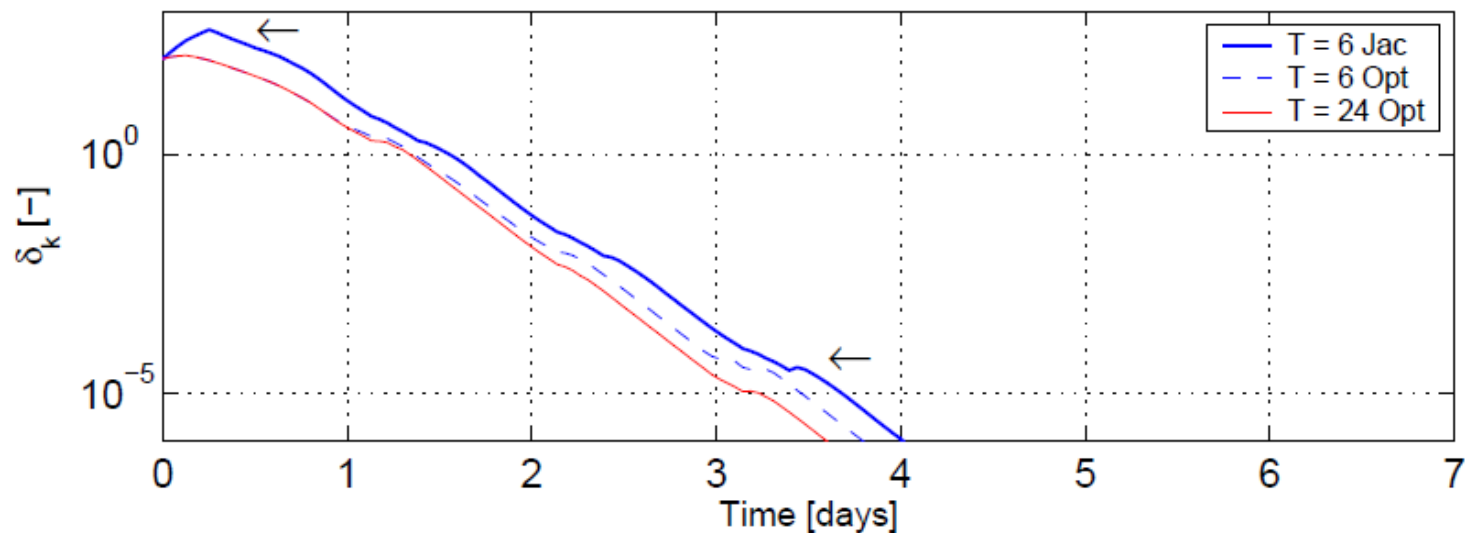
Consider 3 Market Designs

- 6 Hours Horizon, Incomplete Gaming
- 6 Hours Horizon, Complete Gaming
- 24 Hours Horizon, Complete Gaming

Efficiency



Summarizing State

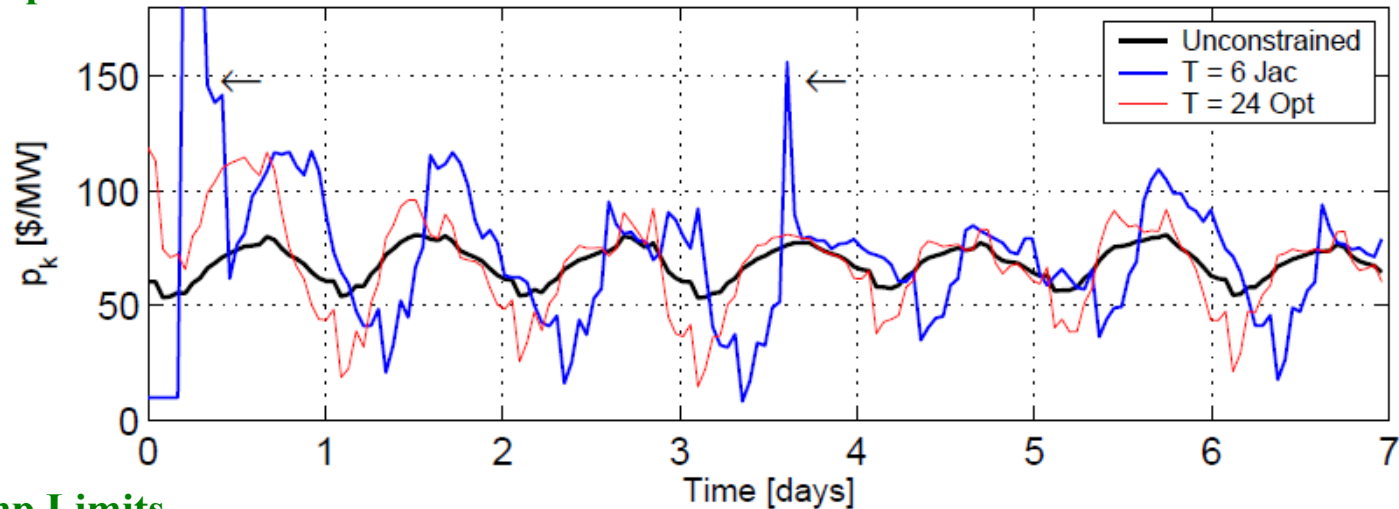


Stability

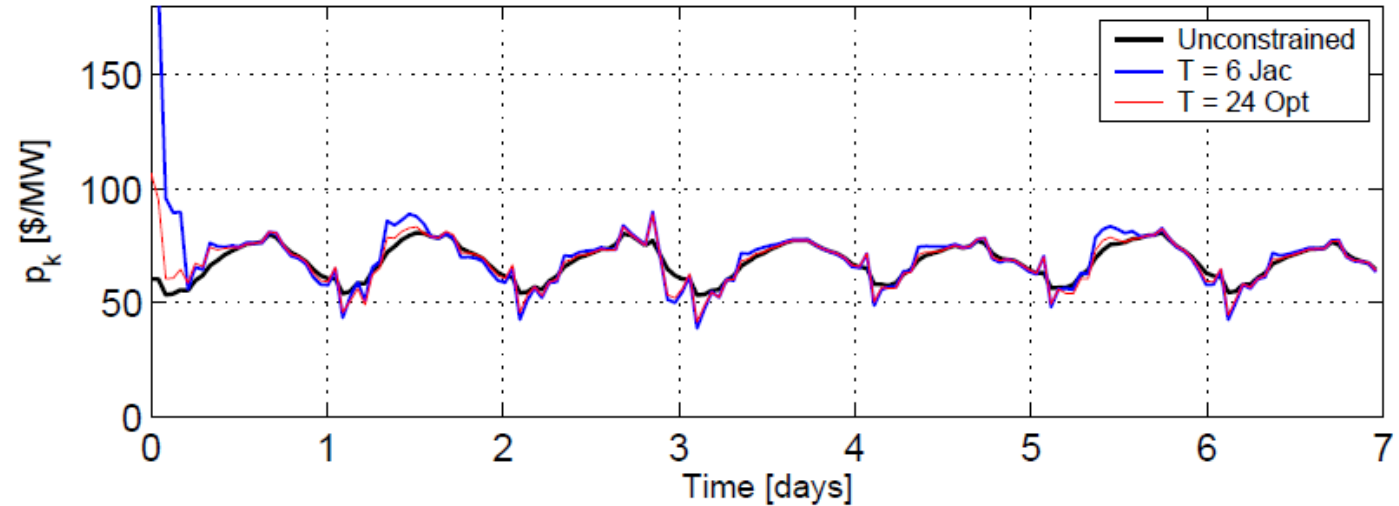
Consider 3 Market Designs

- 6 Hours Horizon, Incomplete Gaming
- 6 Hours Horizon, Complete Gaming
- 24 Hours Horizon, Complete Gaming

Tight Ramp Limits



Lose Ramp Limits



5. Conclusions and Open Questions

Conclusions and Open Questions

Predictive Control Provides a Framework for Market Analysis

- **Advantage: Captures Mechanistic and Physical Effects**
- **Advantage: Captures Decision-Making Rationale (Receding Horizon)**
- **Issue: Market Inherently Dynamic (No Natural Equilibrium)**
- **Issue: Market Stability and Efficiency Definitions are Subjective**

Potential Extensions:

- **Day-Ahead and Real-Time Markets**
- **Stochastic Formulations (Effects of Risk on Stability)**
- **Distributed Optimization Algorithms**
- **Continuous-Time (Closer to Physical Domain)**
- **Alternative Designs (Stabilizing Constraints)**

Alternative Frameworks: Stochastic Stability

Alternative Lyapunov Functions

Differential Variational Inequalities (Existence and Uniqueness)

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